UNIVERSITY OF MANITOBA

ULTRASONICS RESEARCH LABORATORY

Phononic Crystals

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with

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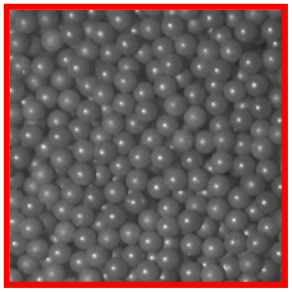
We study ultrasonic waves in complex (strongly scattering) media.

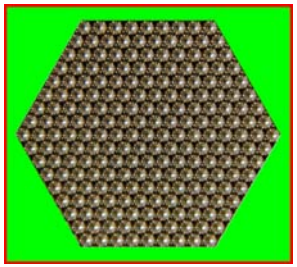


Random media:

- ballistic and diffusive wave transport
- new ultrasound scattering techniques (DSS & DAWS)

Ordered media (phononic crystals):

- ultrasound tunneling
- focusing effects



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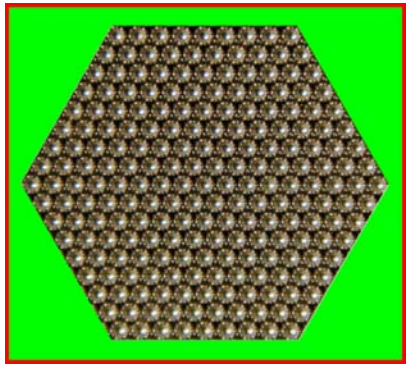
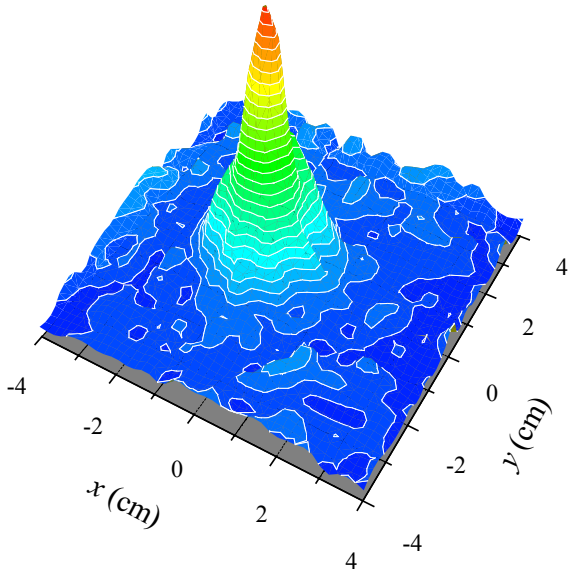
Outline: Phononic Crystals

Motivation: Why study phononic crystals? (acoustic and elastic analogues of photonic crystals):



Wave phenomena in 3D phononic crystals (experiments and theory) :

- our crystals and the experimental setup
- spectral gaps
- ultrasound tunneling
- near field imaging
- focusing effects



Possible applications

- lenses and filters?
- sound insulation?

Conclusions

PHONONIC CRYSTALS

e.g. ordered arrays of mm-sized beads in a liquid or solid matrix $\Rightarrow$ acoustic and elastic analogues of photonic crystals.

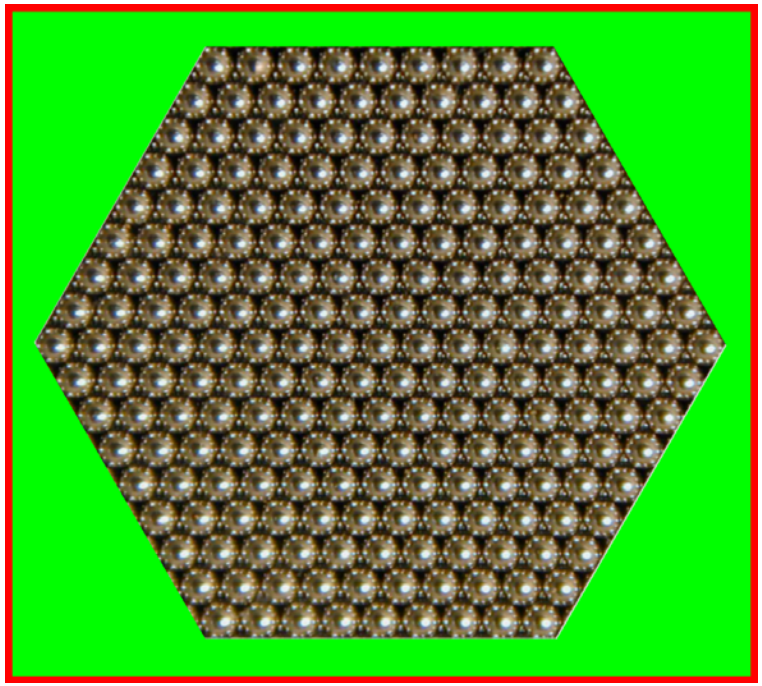
Why study phononic crystals?

- richer physics?
  - scattering contrast $\Leftrightarrow$ $\Delta\rho$, Δv $\rightarrow$ may be easier to achieve complete spectral gaps
  - longitudinal + transverse modes $\Rightarrow$ novel wave phenomena?
- ultrasonic techniques have some advantages
  - measure the field not the intensity
  - pulsed techniques are easy
- good theory is available - e.g. Multiple Scattering Theory for elastic and acoustic waves. [Kafesaki and Economou, PRB **60**, 11993 (1999); Liu *et al*. PRB **62**, 2446 (2000); Psarobas *et al*., Phys. Rev. B **62**, 278 (2000)]
- relatively few experiments have been performed on 3D systems (unlike 2D).
- new applications?

Questions:

- Can complete spectral gaps be readily achieved?
- How do waves travel through phononic band gaps?
  No propagating mode $\rightarrow$ Tunneling?
  How long does it take?
- Can phononic crystals be used to focus ultrasound?

Our 3D Phononic Crystals:



Close-packed periodic arrays of spherical beads surrounded by a liquid or solid matrix:

- hcp arrays of stainless steel balls in water
- fcc arrays of tungsten carbide balls in water
- fcc arrays of tungsten carbide beads in epoxy

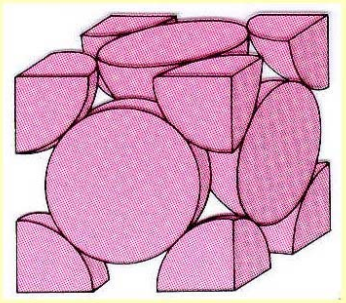
For all these crystals:

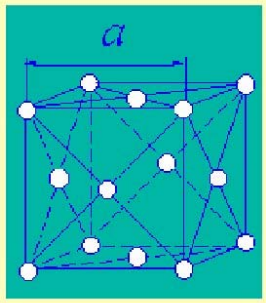
- very high contrast scatterers $Z_{ball}/Z_{matrix} \sim 30$ to 60
- very monodisperse spheres: diameter $d = 0.800 \pm 0.0006$ mm
- very high quality crystals: produced by a manual assembly technique using a hexagonal template.

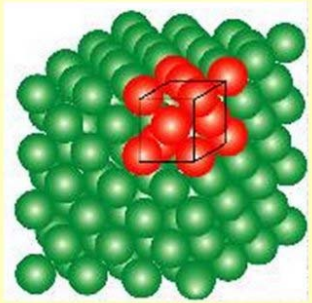
for hcp: ABAB… layer sequence. (slabs with *c*-axis ⊥ layers)

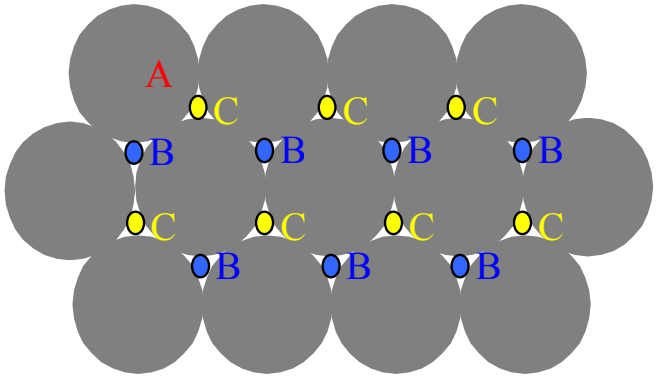
for fcc: ABCABC… layer sequence. ([111]-axis ⊥ layers)

Face-Centred Cubic Crystal Structure









Schematic graph shows how the beads are packed in a FCC crystal along the [111] direction. Beads A are on the bottom layer, B the second and C the third. The sequence is ABCABC...

Material parameters for our phononic crystals made from tungsten carbide beads in water →

Scatterers:

0.800-mm-diameter tungsten carbide beads

$\rho = 13.8$ kg/m^3

$v_L = 6.65$ km/s

$v_T = 3.23$ km/s

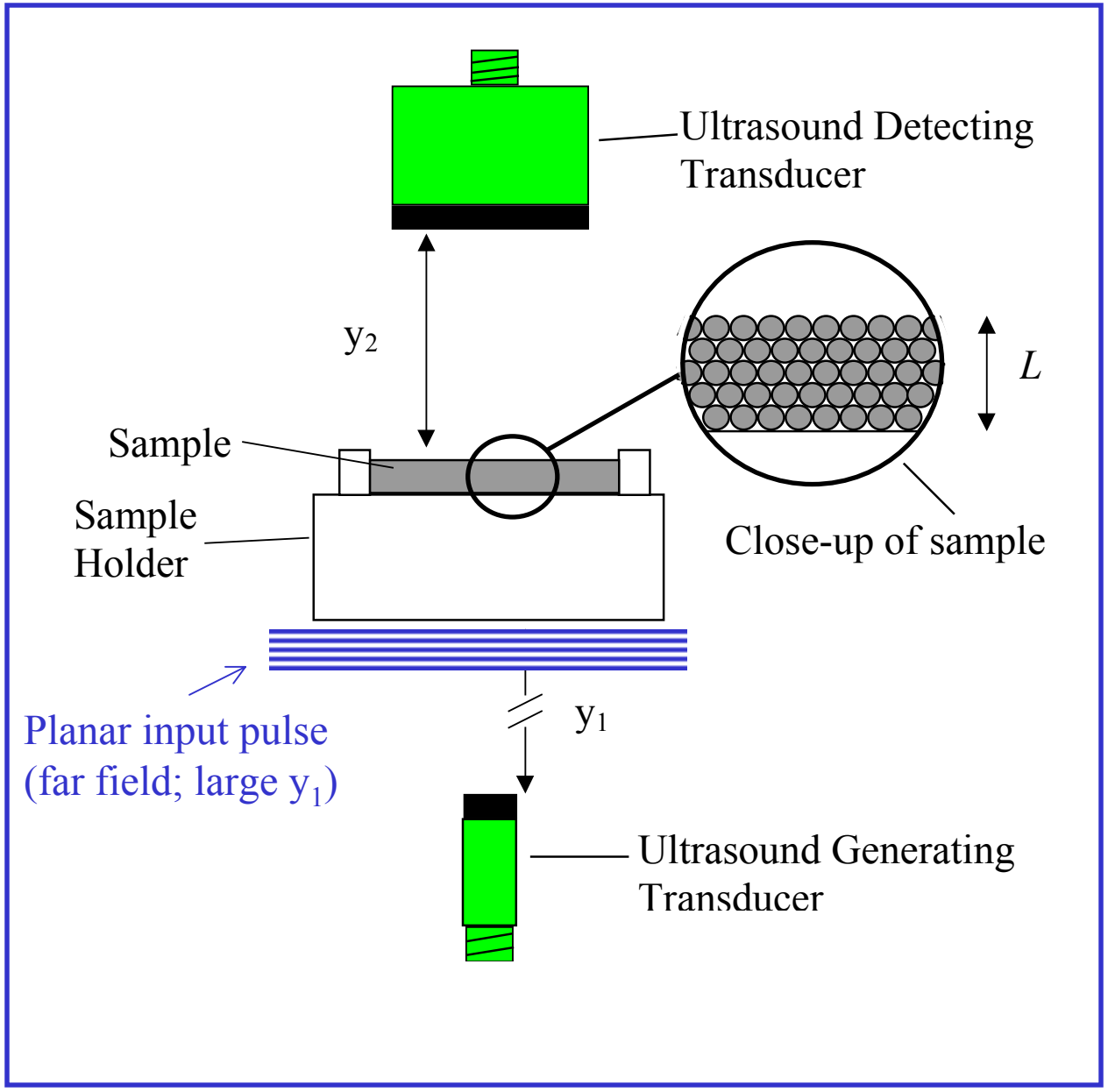
Matrix:

water

$\rho = 1.0$ kg/m^3

$v_L = 1.49$ km/s

Experimental setup



Compare transmitted pulses with reference pulses to measure:

phase velocity:

$v_p(\omega) = \omega L/(\phi + 2\pi n)$

(ϕ = phase delay)

group velocity:

$v_g(\omega) = L/t_{peak}$

(t_{peak} = peak delay time)

transmission coefficient:

$A_{trans}(\omega) / A_{ref}$

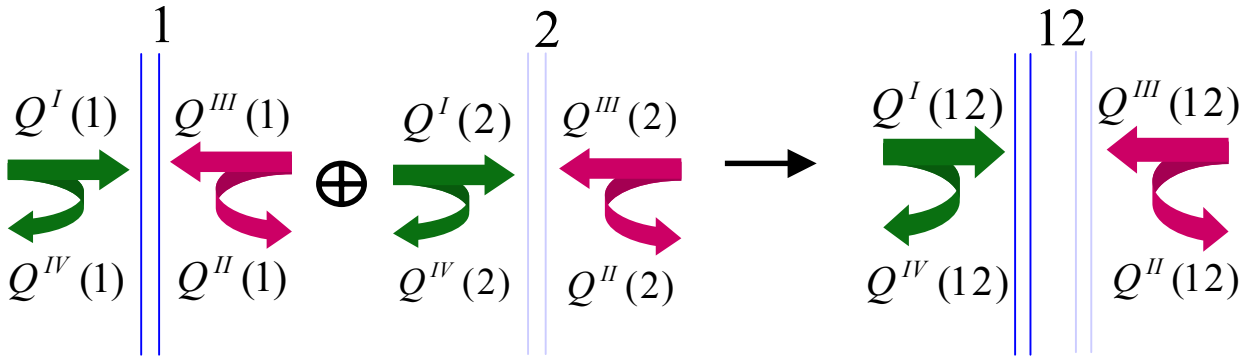
(FFT ratio)

Multiple Scattering Theory

Multiple scattering theory (MST) for acoustic and elastic waves -- ideally suited to spherical scatterers. (cf. KKR theory) [Liu et al. PRB **62**, 2446 (2000)]

Band-structure: calculate elastic Mie scattering of waves from all the scatterers in the crystal, and solve the resulting secular equation for the eigenfrequencies.

Transmission: calculate the transmission through a multi-layer sample with thickness L using a layer MST.



$\Rightarrow$ Transmitted wave $T(L,\omega) = A(L,\omega)\exp[i\phi(L,\omega)]$

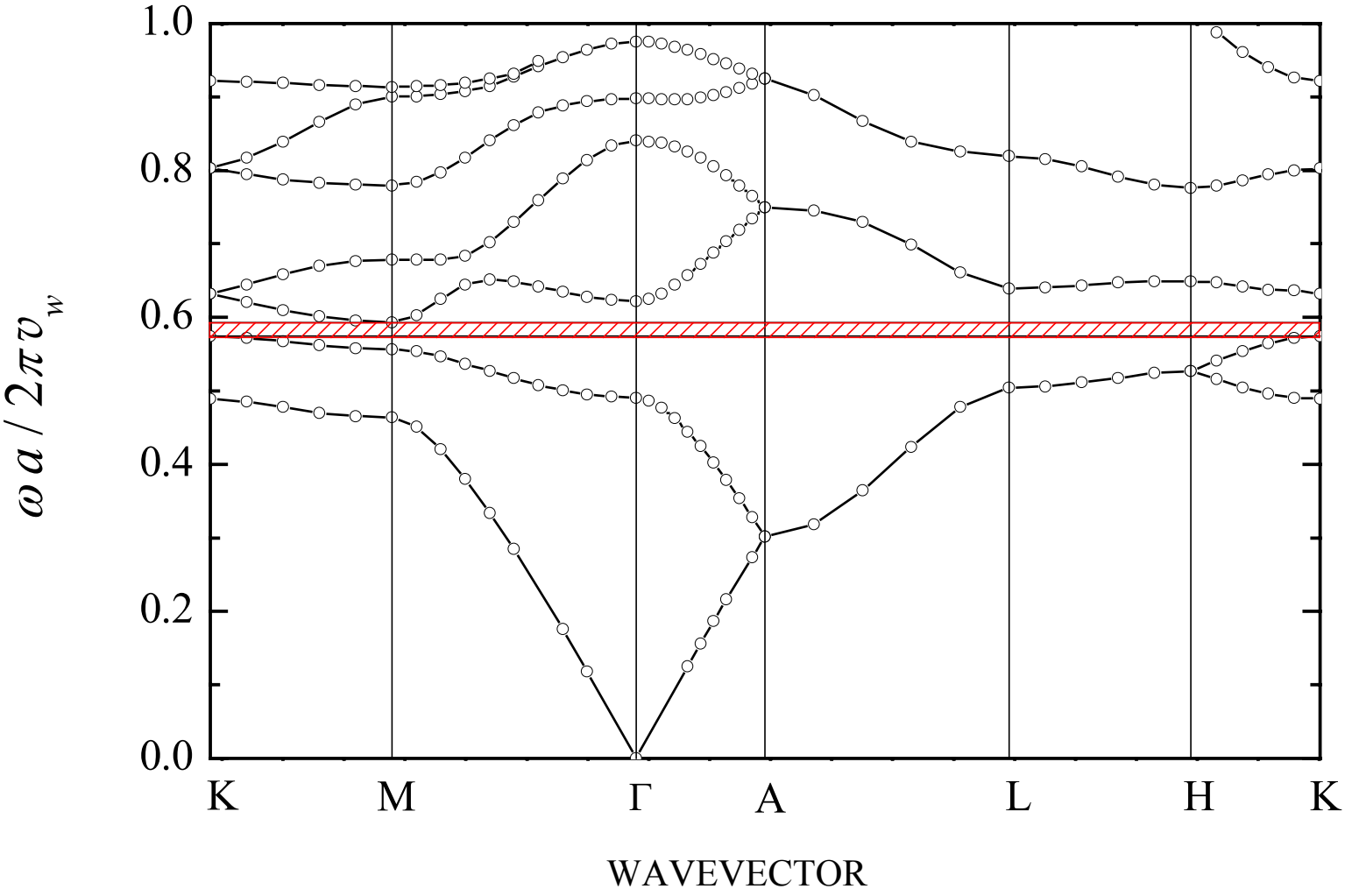
A normalized amplitude (transmission coefficient)

ϕ cumulative phase relative to the incident wave.

Phase velocity $v_p(\omega) = \omega L / \phi$

Group velocity $v_g(\omega) = \dfrac{d\omega}{dk} = L\dfrac{d\omega}{d\phi}$

hcp bandstructure: stainless steel spheres in water

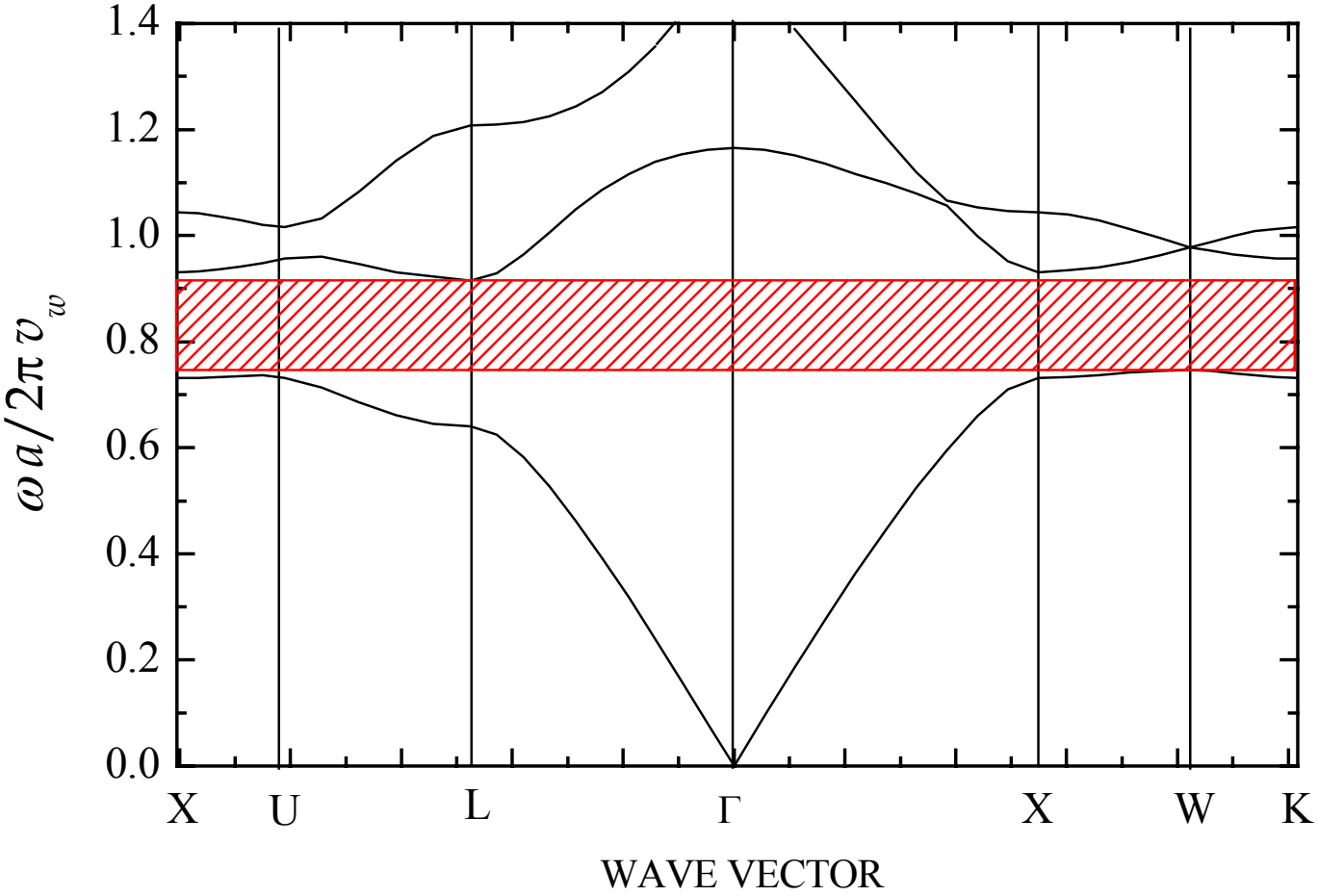


a lattice constant

v_w sound velocity in water.

Very small complete gap

Fcc bandstructure: tunsgsten carbide beads in water

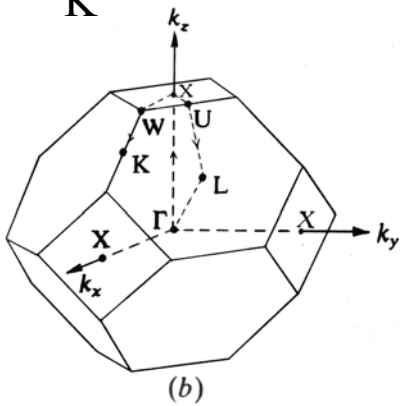


a lattice constant

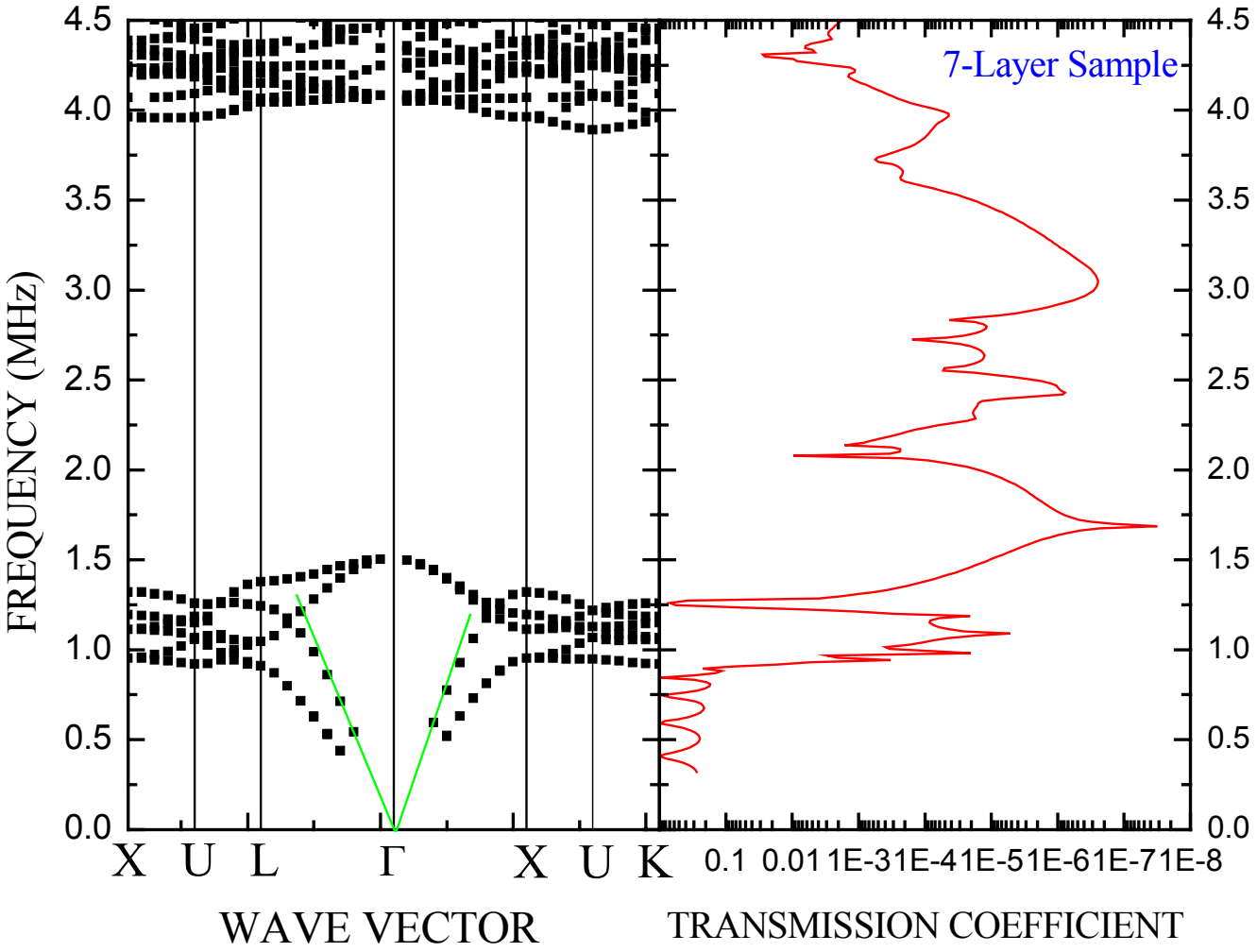
v_w sound velocity in water.

Complete gap: $\Delta\omega/\omega_{\text{centre}} = 19\ \%$

Reciprocal lattice

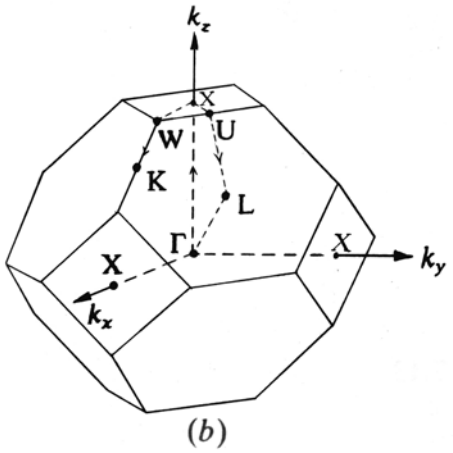


Fcc bandstructure and transmission: tunsgsten carbide beads in epoxy



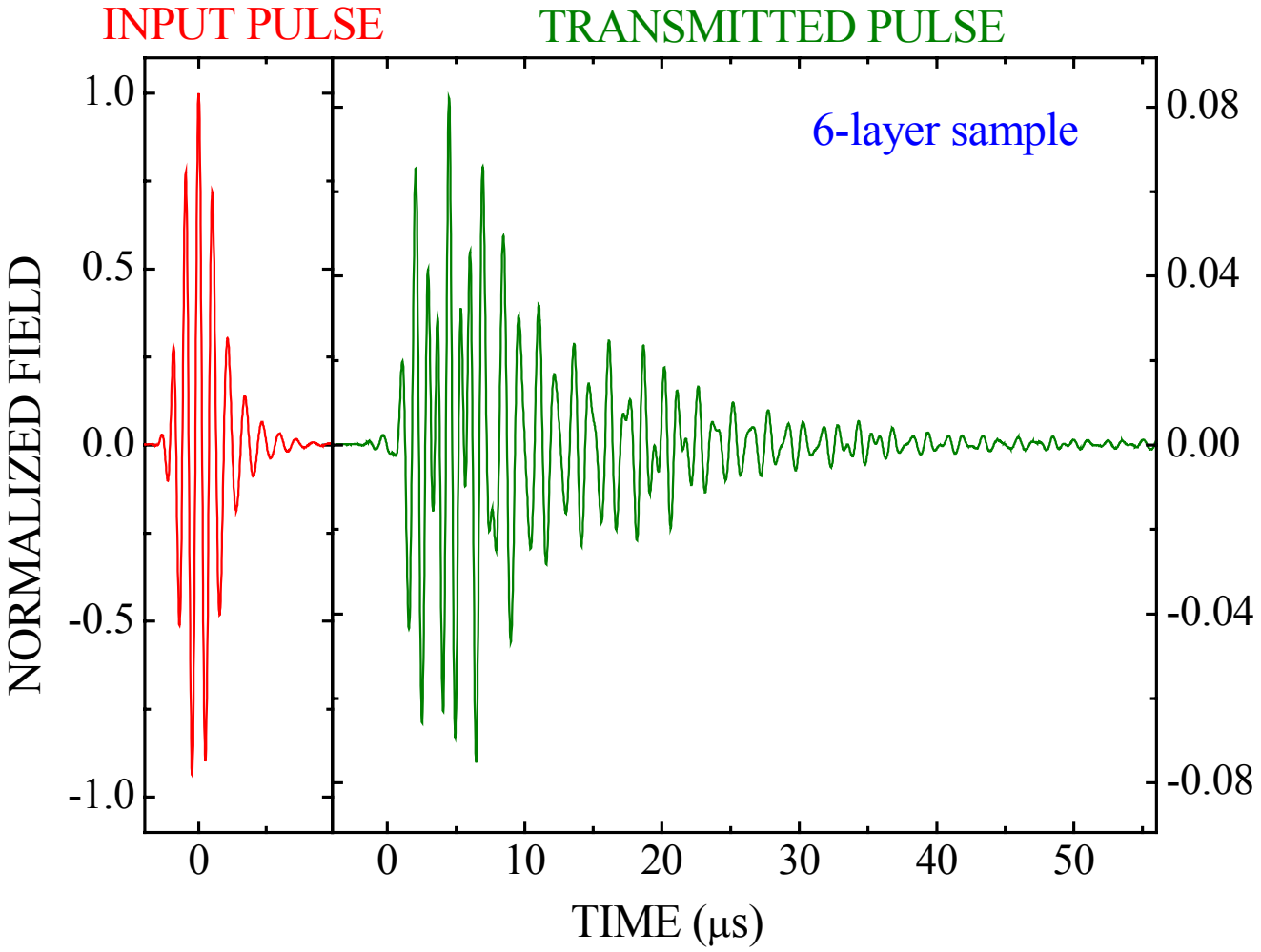
An even bigger complete gap!

$\Delta\omega/\omega_{\text{centre}} = 90\ \%$



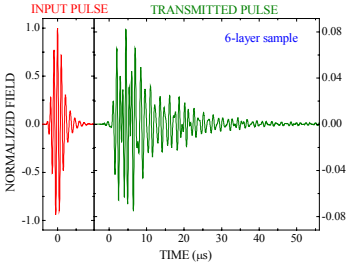
FCC tungsten carbide beads in water:

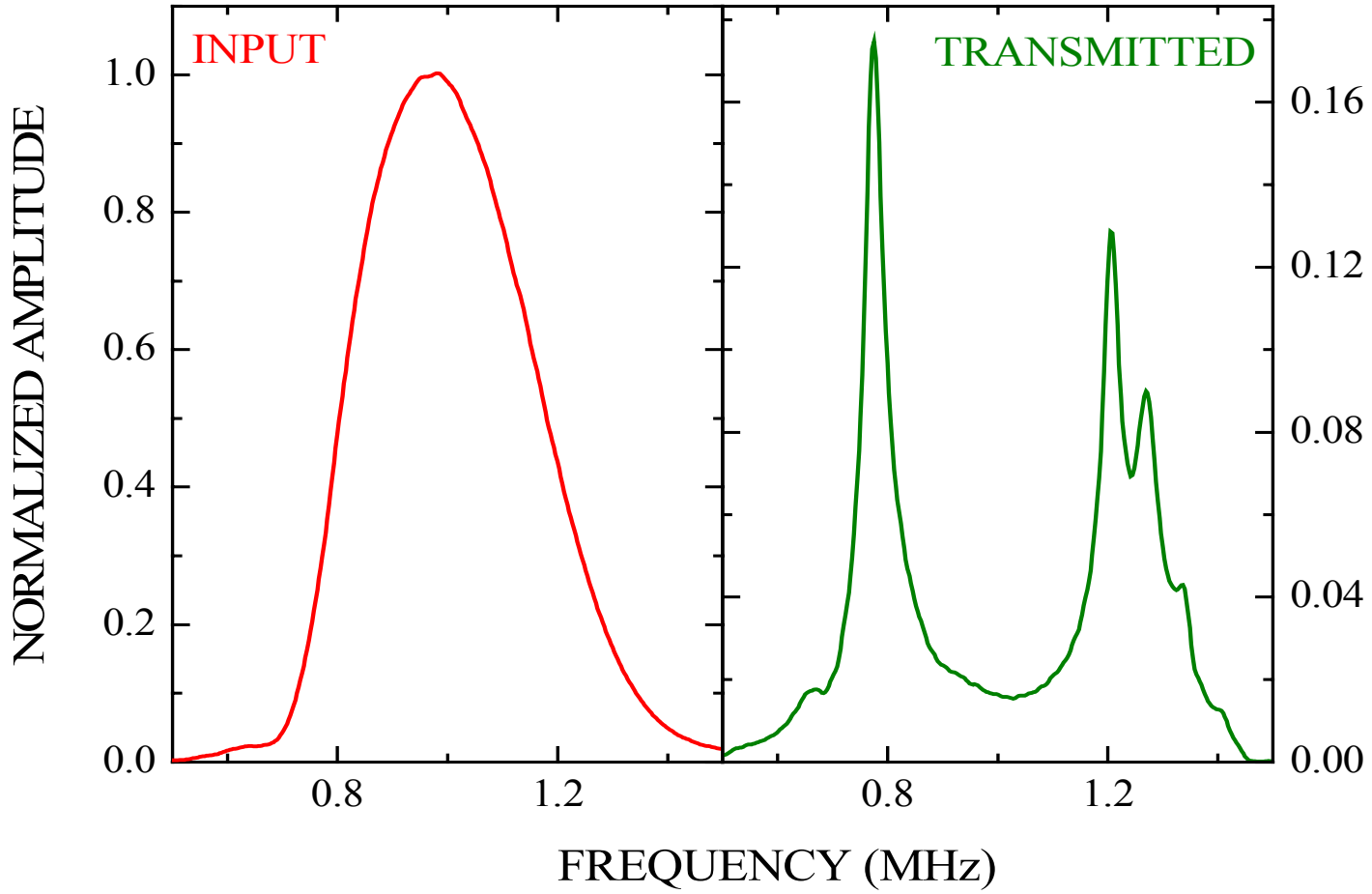
Input and transmitted pulses near the lowest band gap, $k \| [111]$



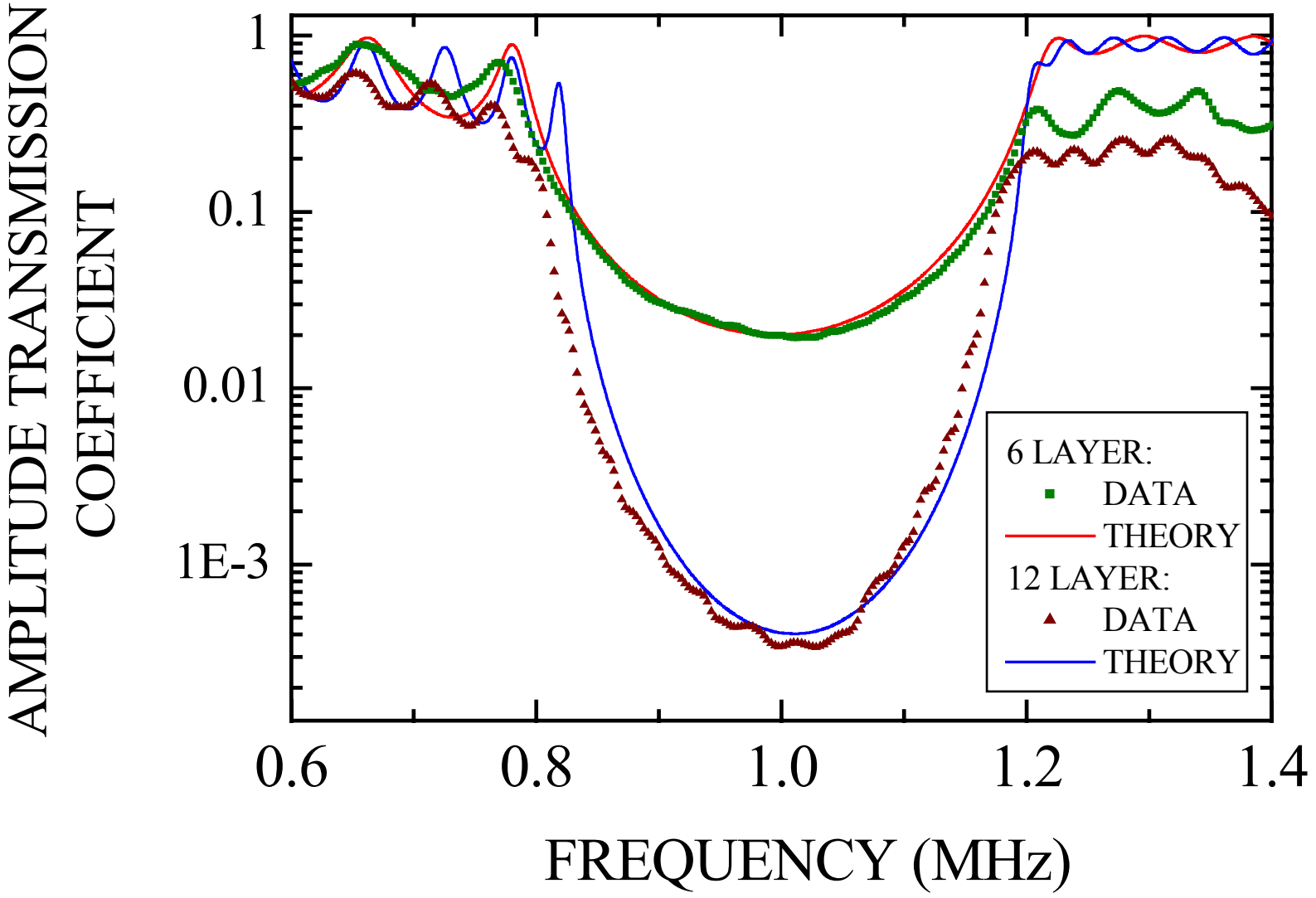
FCC tungsten carbide beads in water:

Fourier Spectrum of these input and transmitted pulses near the lowest band gap



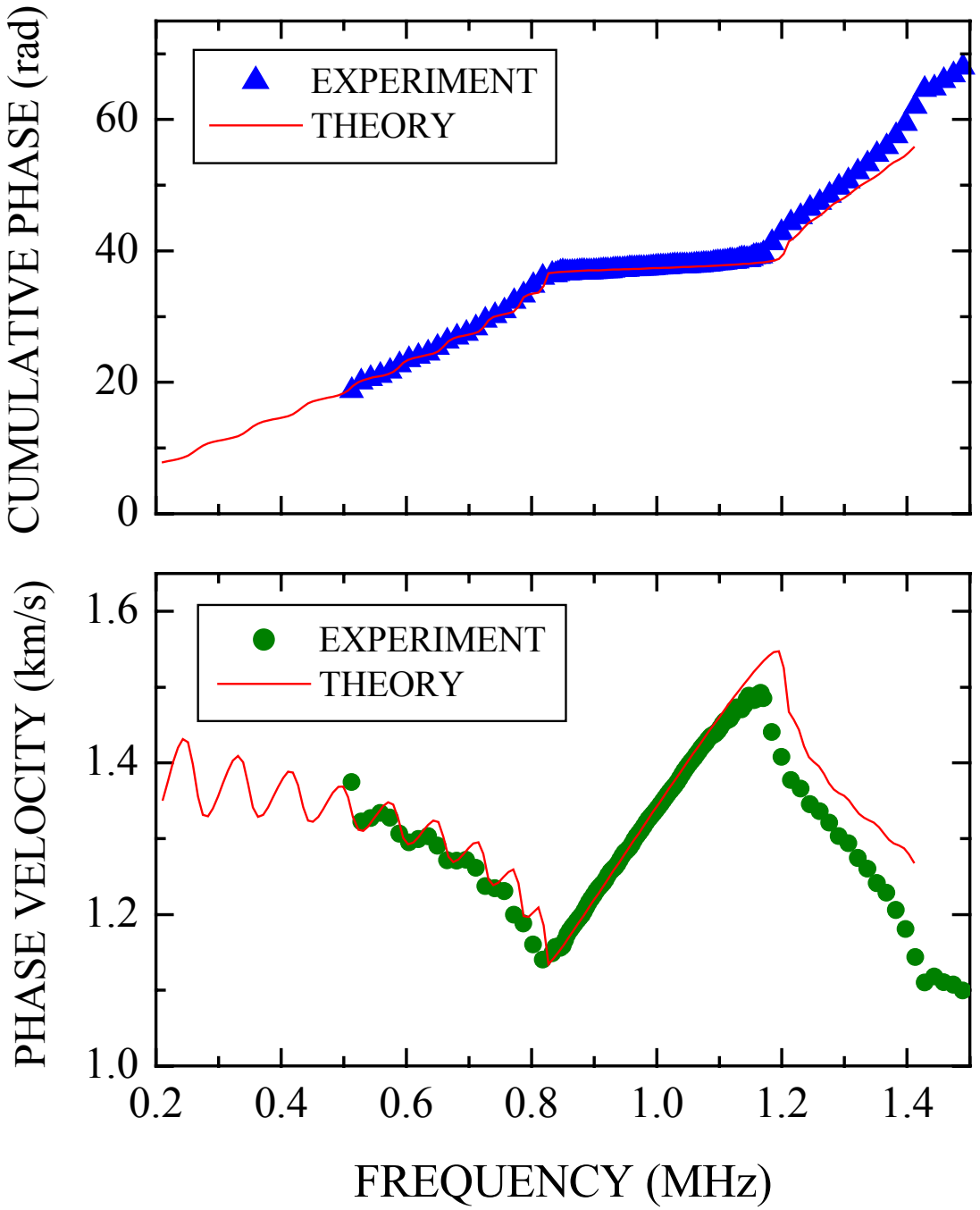


Transmission coefficient



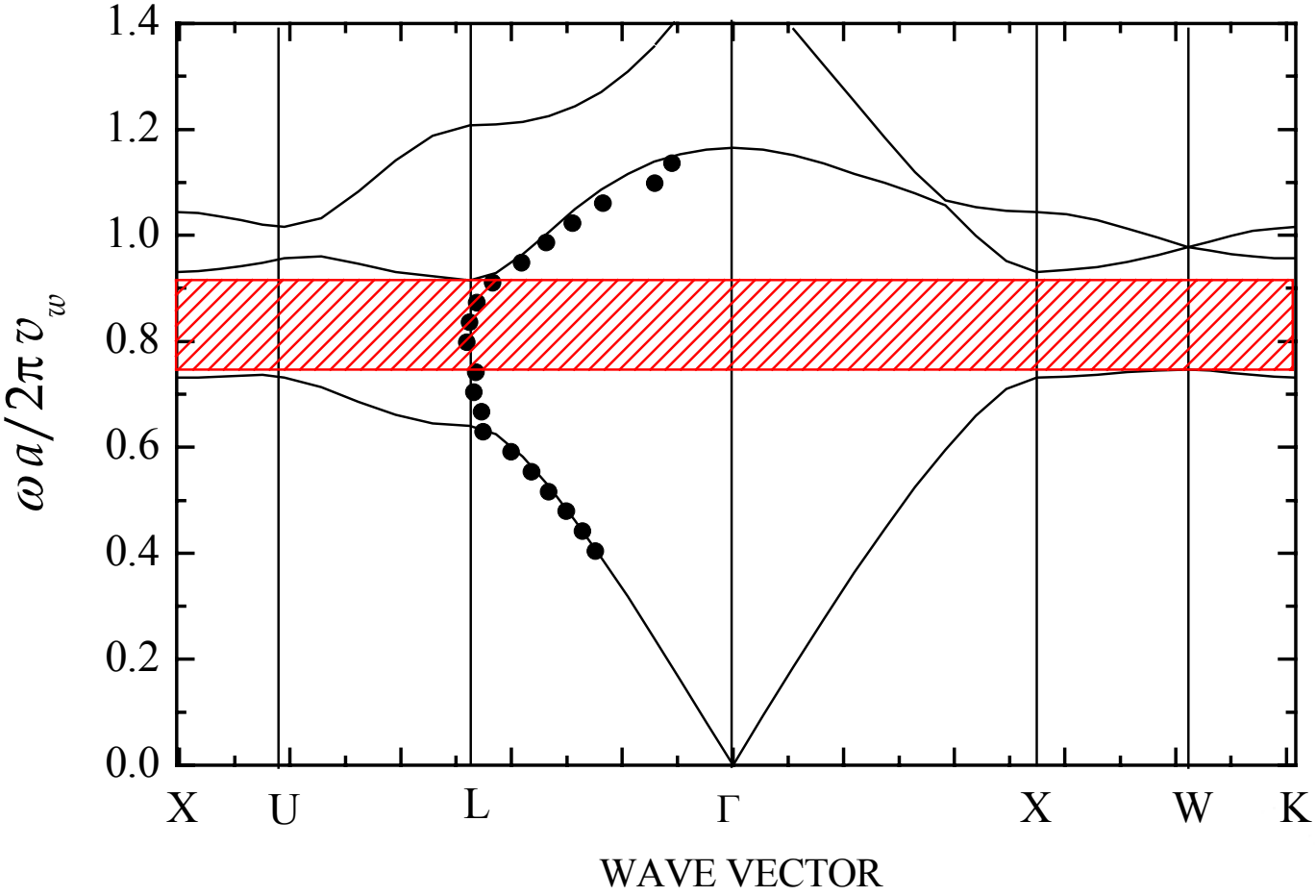
Cumulative Phase and the Phase Velocity

Use these data to compute the dispersion curve, ω versus $k = \omega / v_p$, and compare with the bandstructure calculation…



Fcc bandstructure: tunsgsten carbide beads in water

The black symbols show experimental data from phase velocity measurements



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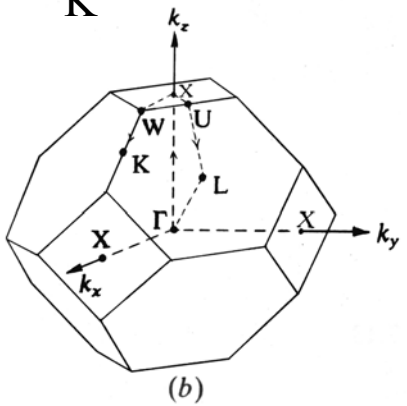
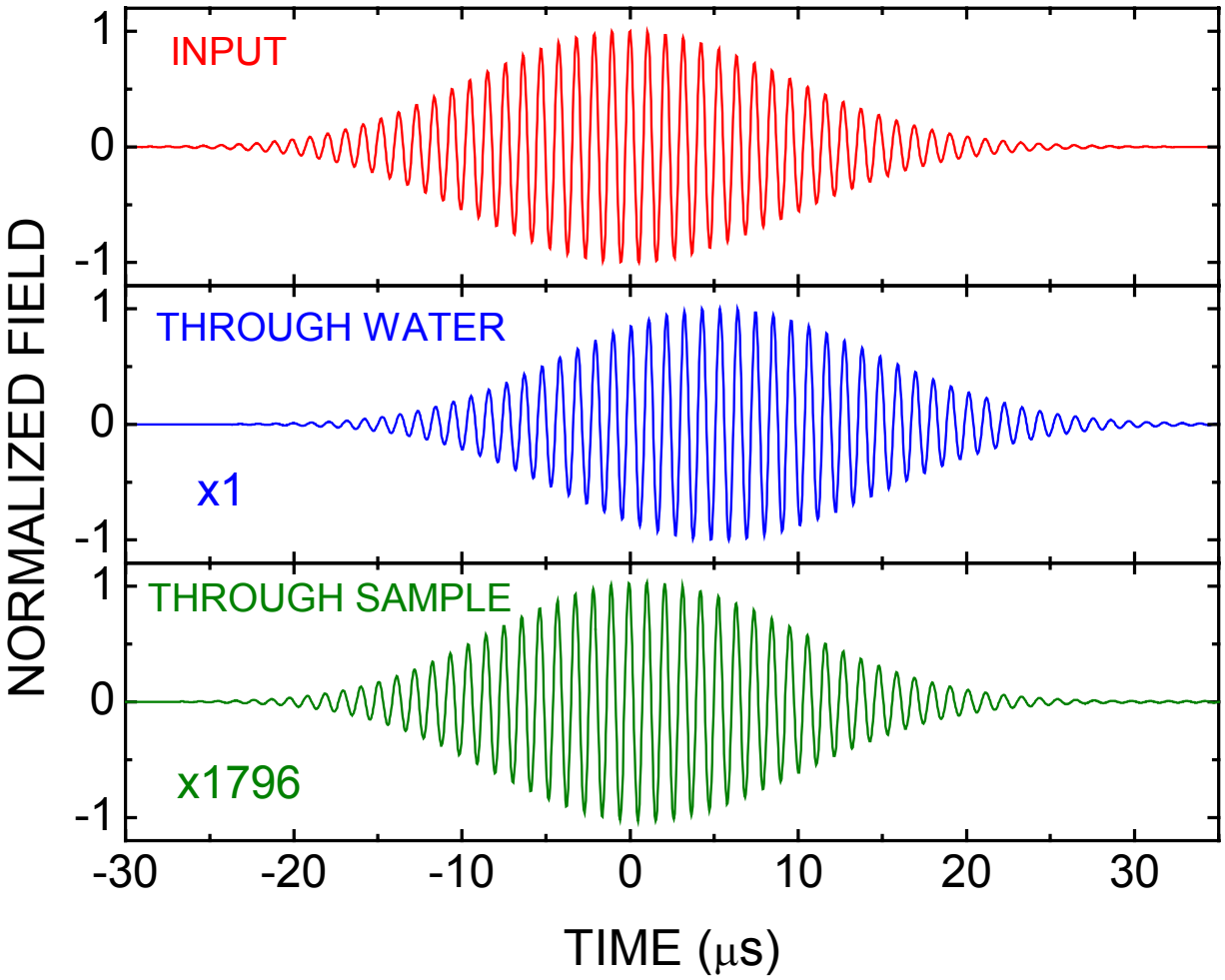
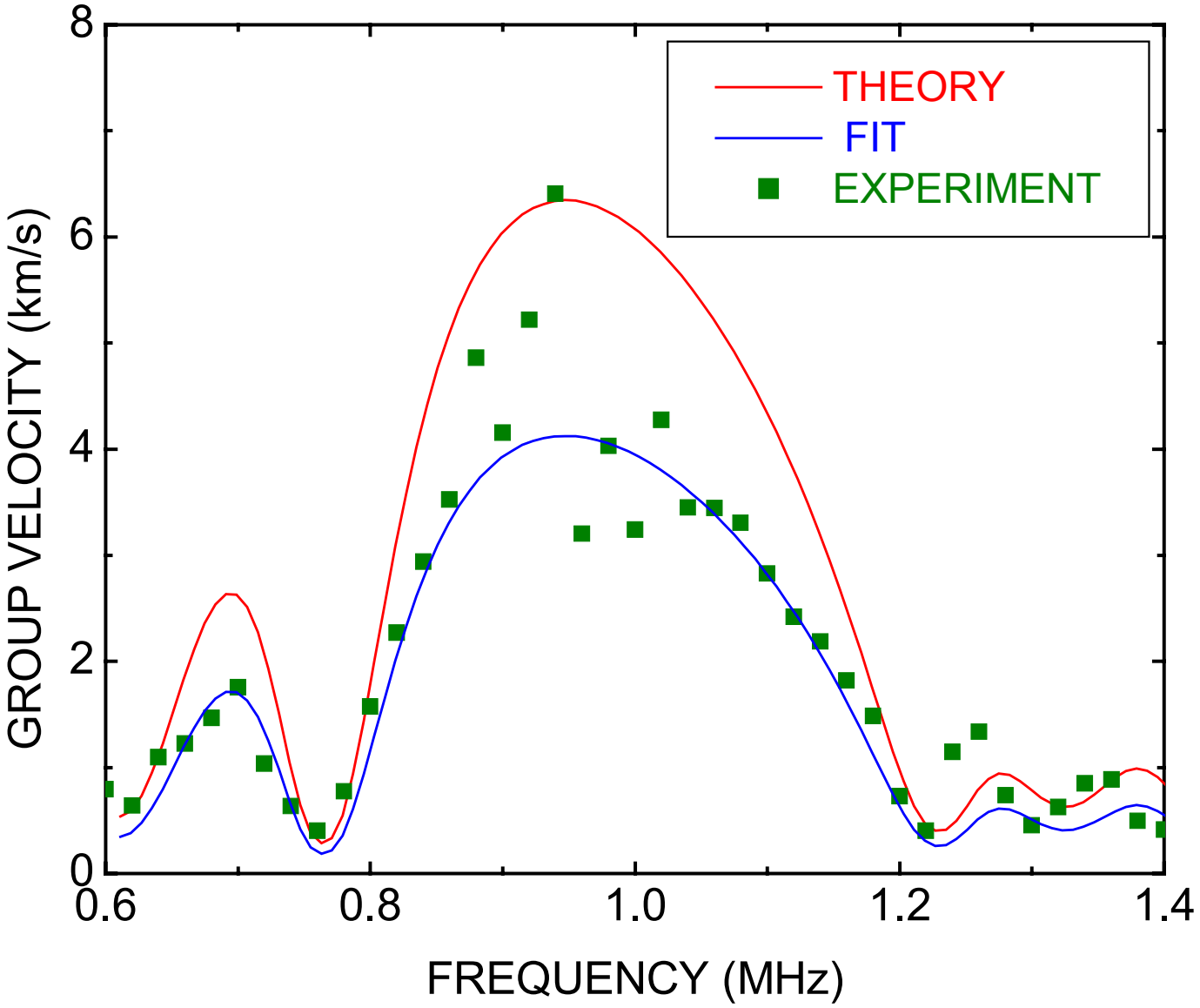


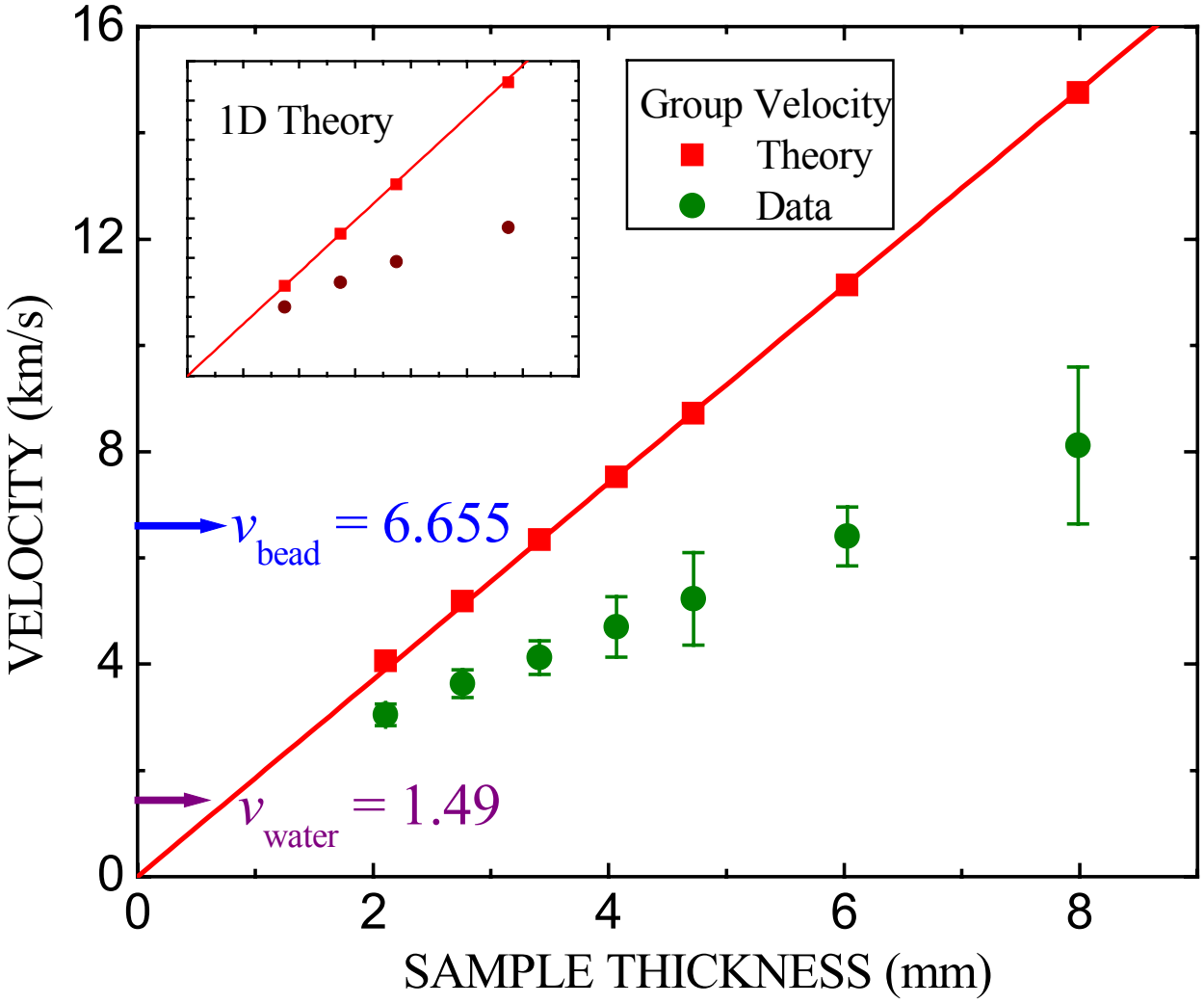
Measuring the group velocity: Digitally filtered input and transmitted pulses (bandwidth 0.05 MHz) for a 12-layer phononic crystal in the middle of the gap, compared with the pulse transmitted through the same thickness of water.



Frequency dependence of the group velocity for a 5-layer sample.



Group Velocity versus sample thickness in the middle of the gap



The group velocity increases linearly with thickness! $\Rightarrow$ tunneling.

Yang *et al.*, Phys. Rev. Lett. **88**, 104301 (2002)

Effect of absorption - the two-modes model

Simple physical picture:

Absorption cuts off the long scattering paths $\Rightarrow$ destructive interference is incomplete.

Effectively, absorption introduces a small additional component having a *real* wave vector inside the gap.

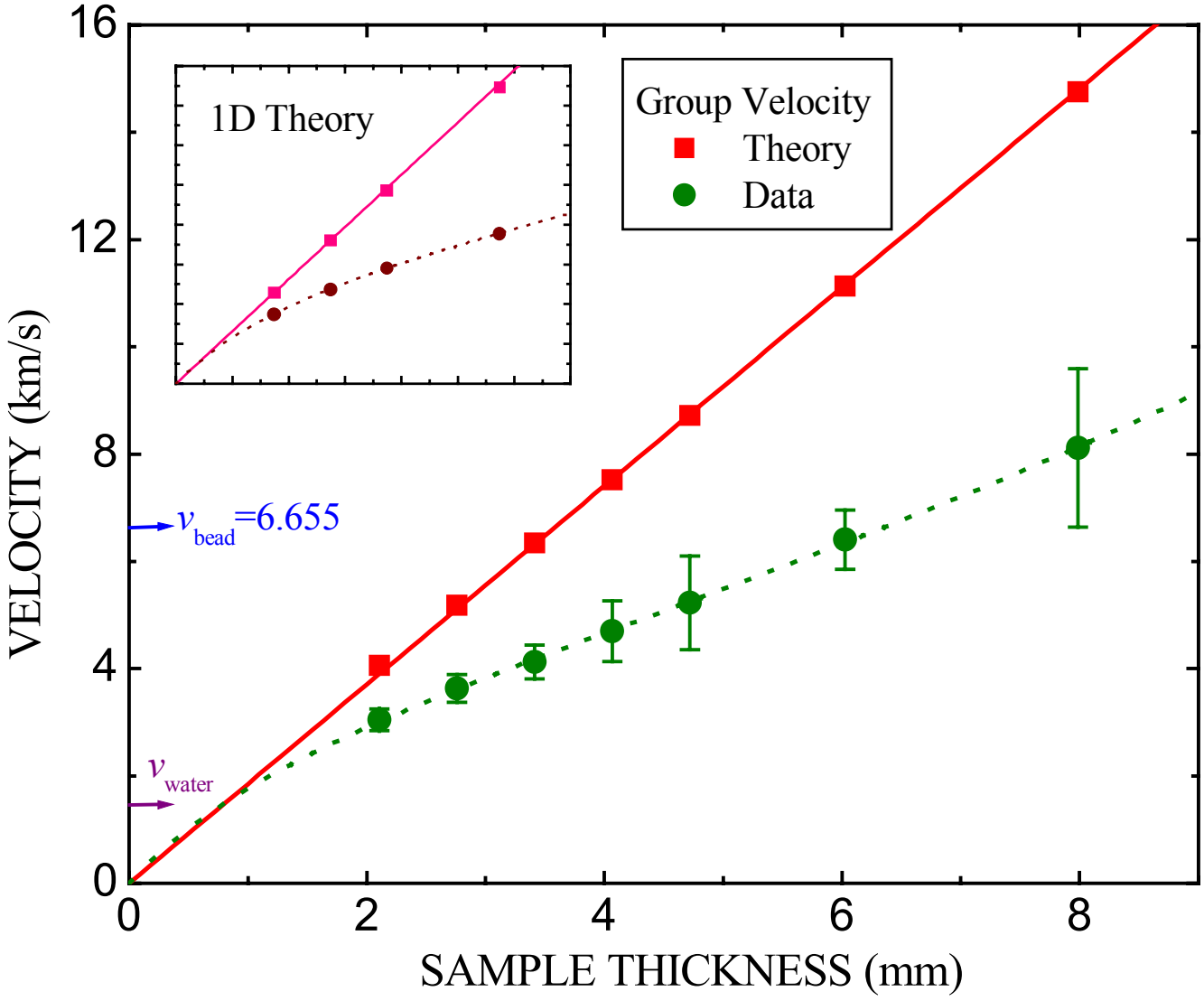
Two-modes model: (Dominant) Tunneling mode (constant tunneling time)
\+ Propagation mode (constant velocity)

$\Rightarrow$ Group velocity becomes

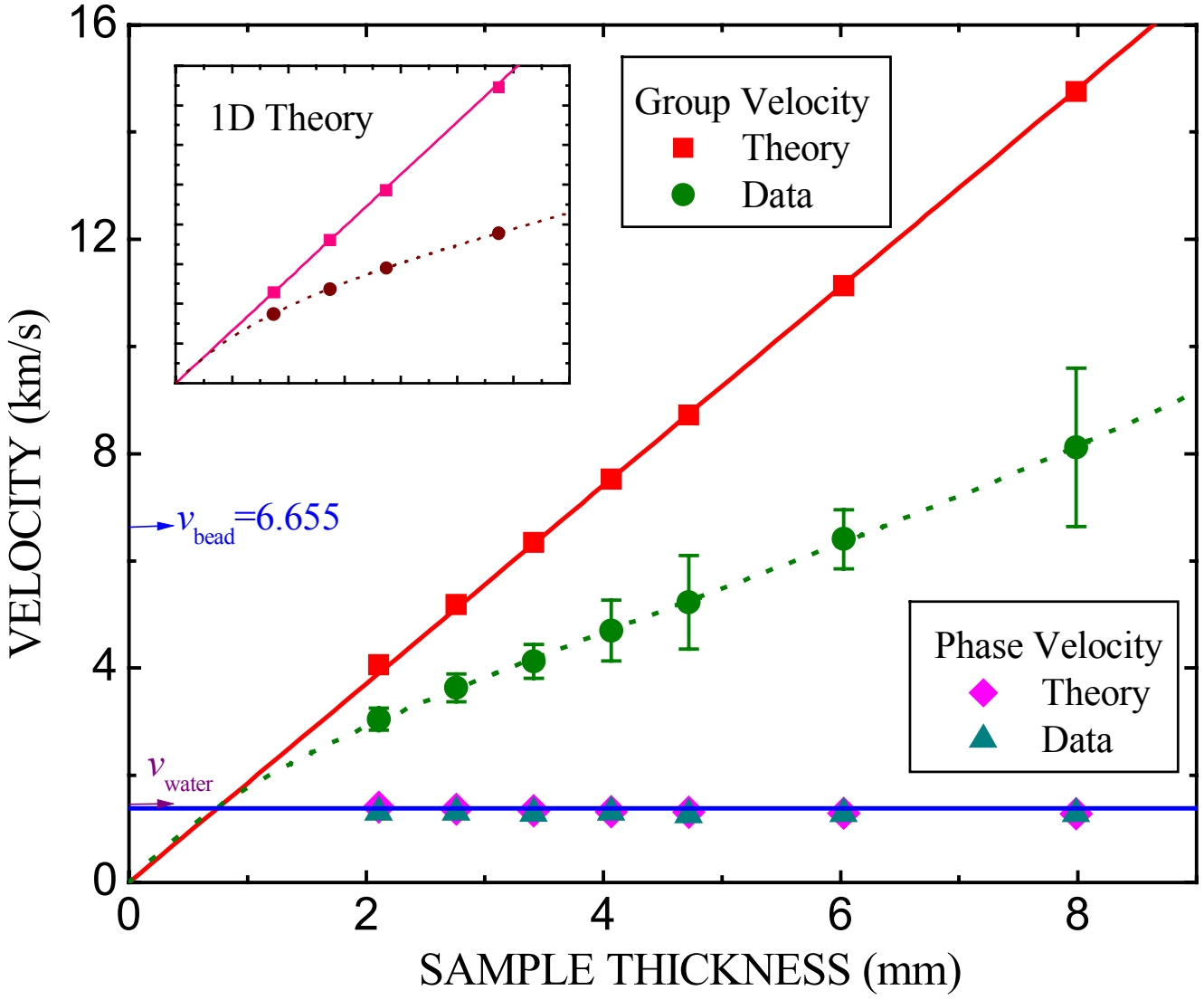
$$\overline{v}_g = \frac{L}{\frac{ae^{-\frac{L}{2l_a}}}{ae^{-\frac{L}{2l_a}}+\sqrt{1-a^2}e^{-\frac{L}{2l_b}}}t_a + \frac{\sqrt{1-a^2}e^{-\frac{L}{2l_b}}}{ae^{-\frac{L}{2l_a}}+\sqrt{1-a^2}e^{-\frac{L}{2l_b}}}\frac{L}{v_b}}$$

where L is the thickness, a is the coupling coefficient, t_a and l_a are the tunneling time and tunneling length, v_b is the group velocity of the propagating mode and l_b is its decay length.

The effect of absorption: Fits of the two-modes model to 1 D theory and to our data for 3 D phononic crystals (dashed curves).



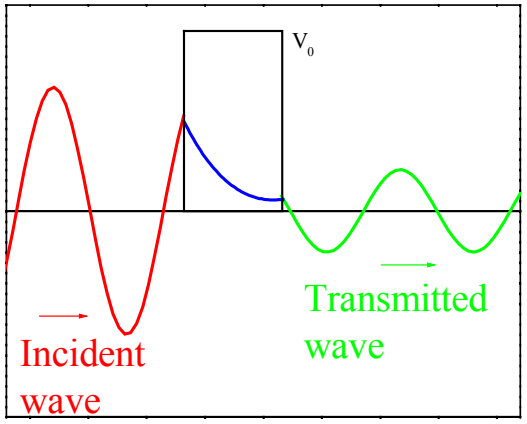
Add: the phase velocity in the middle of the gap for the same sample thicknesses.



Electron tunneling through a barrier

Schrodinger equ.

$$-\frac{\hbar^2}{2m}\frac{d^2\varphi}{dx^2} = -(V-E)\varphi(x)$$



Boundary conditions

$$\varphi(0^-) = \varphi(0^+); \quad \varphi'(0^-) = \varphi'(0^+)$$

$$\varphi(L^-) = \varphi(L^+); \quad \varphi'(L^-) = \varphi'(L^+)$$

$$V = \begin{cases} 0 & x < 0 \\ V_0 & 0 \le x \le L \\ 0 & x > L \end{cases}$$

From the transmitted phase at large large L, find

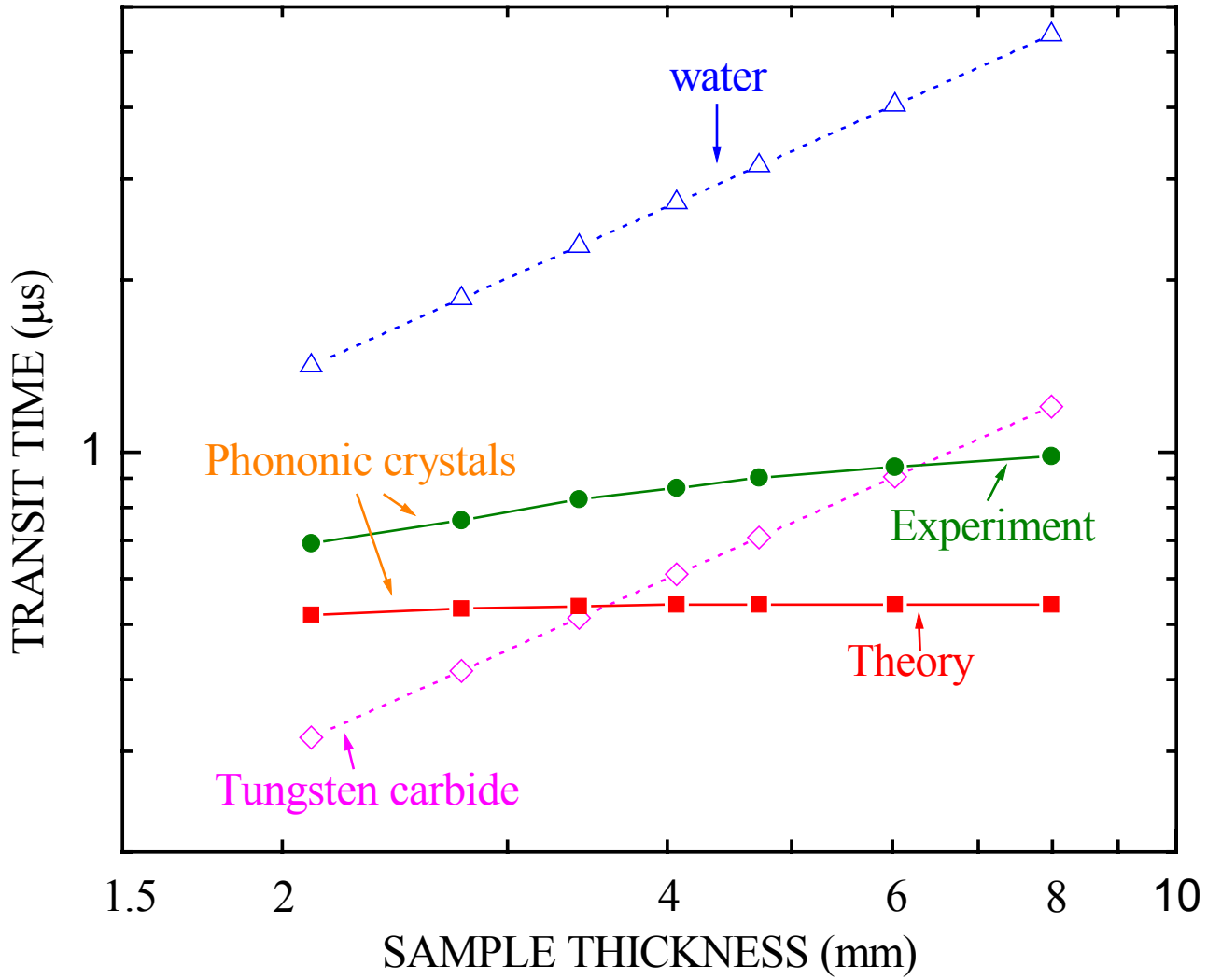
$$v_p = L\bullet\omega \Big/ \arctan\left[\frac{(2\hbar\omega - V_0)}{2\sqrt{(V_0-\hbar\omega)\hbar\omega}}\right]$$

$$v_g = L\bullet\sqrt{\left(\frac{V_0}{\hbar}-\omega\right)\omega}$$

Barrier: Both phase and group velocities increase with L.

Band gap: Group velocity increases while the phase velocity is constant.

Tunneling Time



Magnitude of the tunneling time: $t_{\text{tunnel}} \sim 1 / \Delta\omega_{\text{gap}}$

(also true for light and electrons!)

Question: Is ultrasound tunneling dispersionless?

Some general points about pulse propagation in dispersive media:

Incident pulse at $x = 0$ $\rightarrow$ Fourier transform:

$$\psi(x=0,t) = e(t)\exp\left[i\omega_o t\right] \qquad \Psi(\omega) = \Im\left[\psi(t)\right] = E(\omega-\omega_o)$$

$e(t)$: pulse envelope

$E(\omega)$ is F.T. of $e(t)$

Pulse after traveling a distance x

$$\psi(x,t) = \Im^{-1}\left[\exp\left[-\frac{x}{2l_e}\right]E(\omega-\omega_o)\exp\left[-ik(\omega)x\right]\right]$$

extinction length, $l_e^{-1} = l_s^{-1} + l_a^{-1}$

l_s: scattering l_a: absorption

Expand the wave vector $k(\omega)$ in a Taylor series about the central frequency ω_o:

$$k(\omega) = k_o + \left.\frac{dk}{d\omega}\right|_o \{\omega-\omega_o\} + \tfrac{1}{2!}\left.\frac{d^2k}{d\omega^2}\right|_o \{\omega-\omega_o\}^2 + \ldots$$

inverse group velocity

group velocity dispersion (GVD)

Pulse at (x,t) becomes

$$\psi(x,t) = \exp\left\{-\frac{x}{2l_e}\right\}\exp\{-ik_o x\}\cdot$$

$$\Im^{-1}\left[E(\omega-\omega_o)\exp\left\{-i\left(dk/d\omega\right)\Big|_o\{\omega-\omega_o\}x\right\}\exp\left\{-i\tfrac{1}{2}\left(d^2k/d\omega^2\right)\Big|_o\{\omega-\omega_o\}^2 x\right\}\ldots\right]$$

$$\cong \exp\left\{-\frac{x}{2l_e}\right\}\exp\{-ik_o x\}\exp\{i\omega_o t\}\,\Im^{-1}\left[E(\omega)\exp\left\{-i\left(dk/d\omega\right)\Big|_o x\omega\right\}\exp\left\{-i\tfrac{1}{2}\left(d^2k/d\omega^2\right)\Big|_o x\omega^2\right\}\right]$$

$$= \exp\left\{-\frac{x}{2l_e}\right\}\exp\left[i\omega_o\left\{t-\frac{x}{v_p}\right\}\right]\left\{\delta\left(t-\frac{x}{v_g}\right)\oplus e(t)\oplus\Im^{-1}\left[\exp\left\{-i\tfrac{1}{2}\left(d^2k/d\omega^2\right)\Big|_o x\omega^2\right\}\right]\right\}$$

convolution

attenuation

carrier frequency oscillations travel at $v_p = \omega_o/k_o$

peak travels at the group velocity of the central frequency

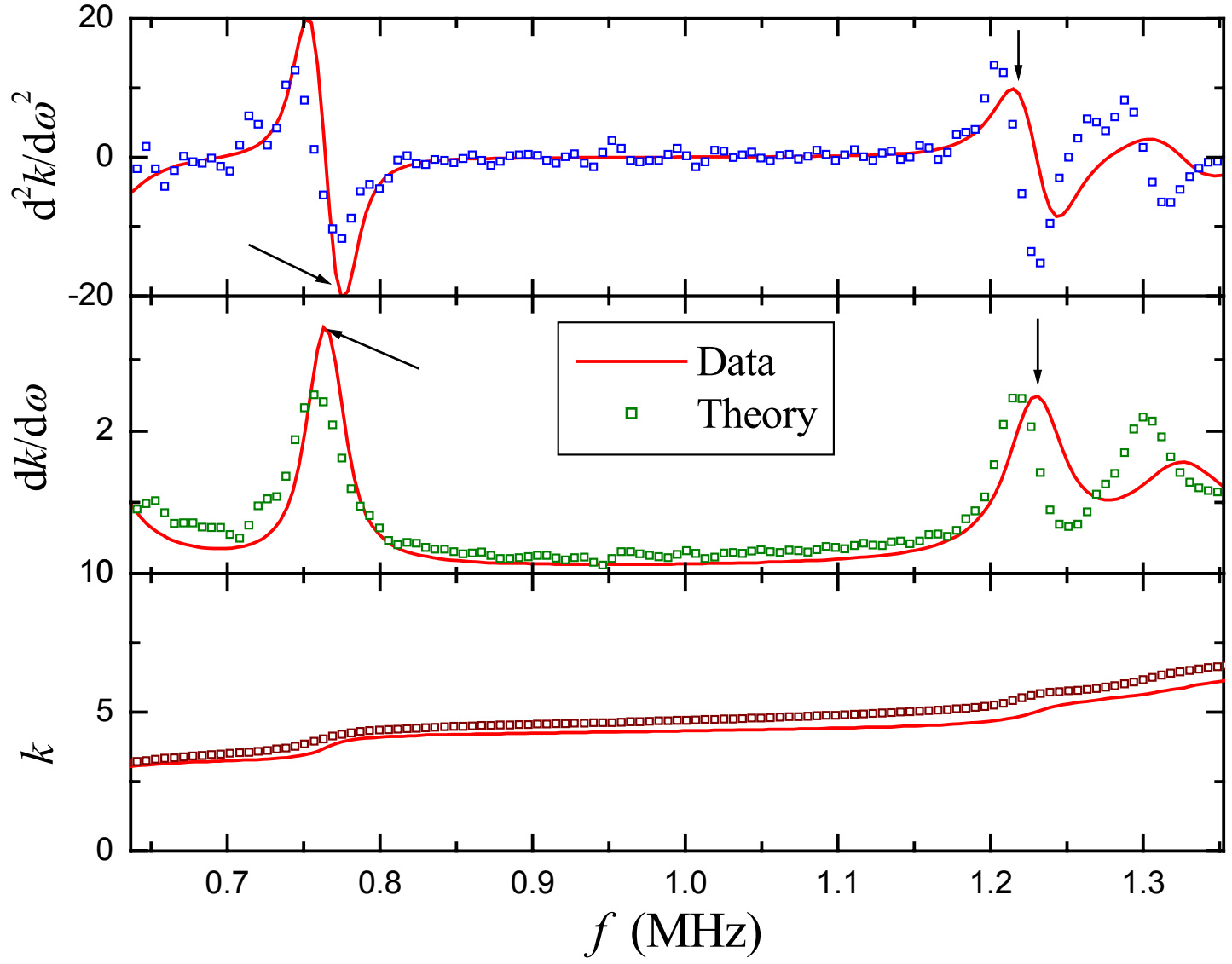
GVD changes the width symmetrically

Good approximation, so long as the pulse bandwidth is sufficiently narrow that:

- higher order terms in $k(\omega)$ expansion are negligible
- frequency dependence of l_e is not important.

Frequency dependence of the wave vector, the inverse group velocity and the group velocity dispersion near the band gap in a 5 layer tungsten carbide/water crystal.

[c.f. measurements near the band edge in photonic crystals by Imhof *et al.* PRL **83**, 2942 (1999)]



Pulse broadening (a) just above the band gap, (b) in the middle of the gap and (c) just below the band gap (bandwidth 0.02 MHz). ⇒ Very weak dispersion in the gap, despite very strong scattering and a large variation in v_p with ω.